How to Teach and Assess Multiplicative Thinking in Primary and Lower Secondary School

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Sources :

Department of Education and Training, State of Victoria, Australia
https://www.education.vic.gov.au

Reyes, R., Lindquist, M., Lamdin, D., Smith, M., Rogers, A., Cooke,A., Ewing, B., Robson, K., Bennet, S. (2016). Helping children learn mathematics (2nd Australian Edition). Wiley.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2011). *Teaching mathematics: Foundations to middle years*. Oxford University Press.

Introductory Comment: Coping with the Pandemic, calculating probability and risk = coping with anxiety successfully

Acknowledgement Of Country

- … let us take a moment to reflect the meaning of place and doing so recognise the various traditional lands on which we do our business today. We acknowledge the Elders – past, present and emerging of all the land we work and live on and their Ancestral Spirits with gratitude and respect.

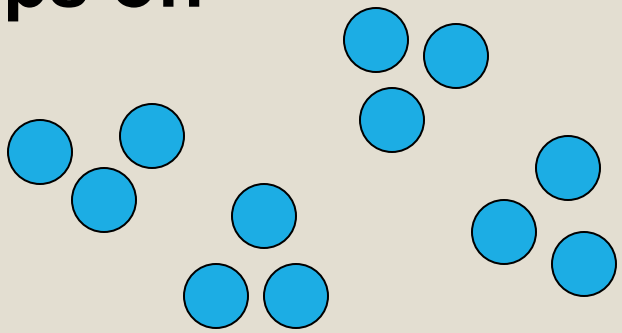
- Deakin would like to Acknowledge the Traditional Custodians of the land on which our Deakin University campuses reside. The Wadawurrung people of the Kulin Nation on whose land our Geelong campuses are located, the Wurundjeri people of the Kulin Nation on whose land our Burwood campus is located and the Gunditjmara people on whose land our Warrnambool campus is located.

- We also pay our respects to all Aboriginal and Torres Strait Islander students who join us at Deakin University and will always be mindful and respectful that our First Nations people are the Traditional Custodians of this country.

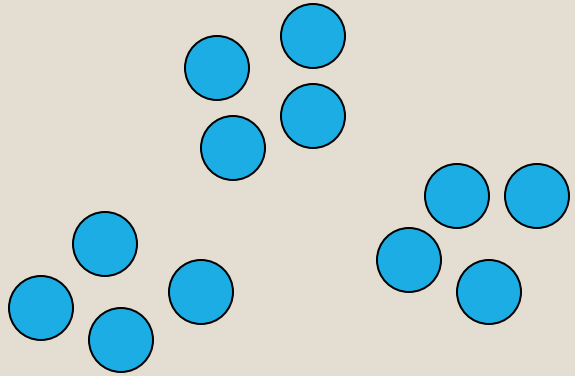
USEFUL CONCEPTS FOR MULTIPLICATIVE THINKING - THE ‘GROUPS OF’ MODEL

CONCEPTS FOR MULTIPLICATION:

1. Groups of:



4 threes ... 3, 6, 9, 12



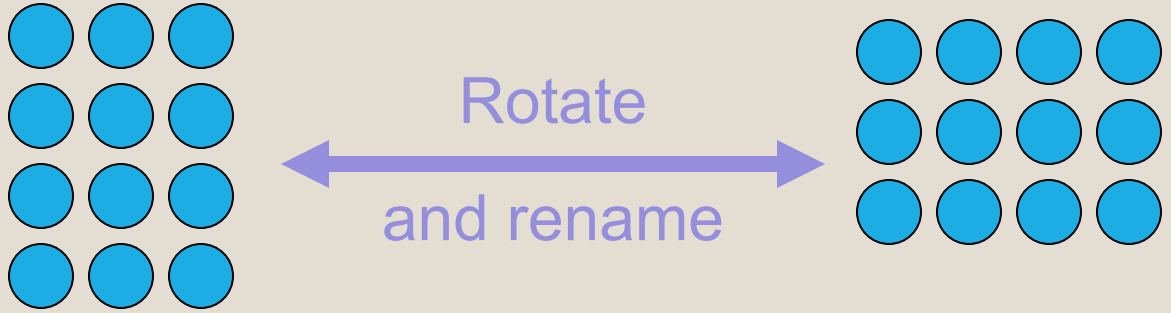
3 fours ... 4, 8, 12

Focus is on the group. Really only suitable for small whole numbers

Strategies: make-all/count-all groups, repeated addition (or skip counting).

MULTIPLICATIVE THINKING -THE ‘ARRAYS’ MODEL

2. Arrays:



4 threes

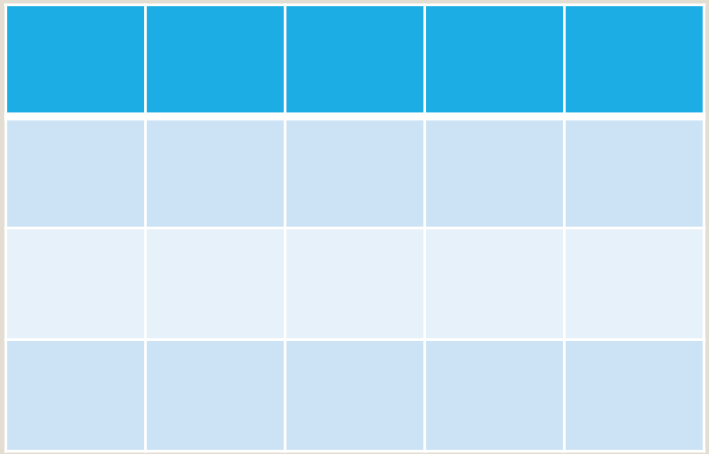
3 fours

Arrays support commutativity

and lead to more efficient mental strategies

MULTIPLICATIVE THINKING -THE ‘REGION’ MODEL

2. Region: e.g. 4 x 5



Note the colours in this table don't have any meaning, they are just unavoidable with this software

Use the game 'Multiplication Toss' to reinforce flexibility with number
e.g. 4 fives
=2 fives + 2 fives
= 3 fives + 1 five etc

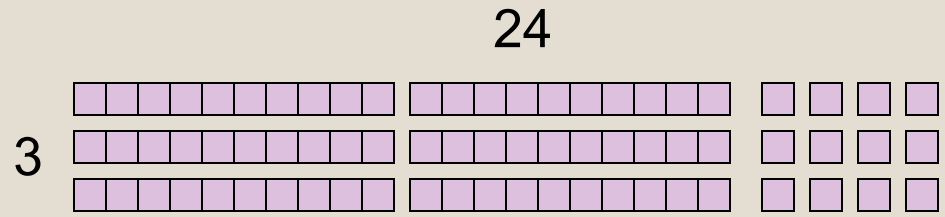
https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/assessment/multiplicationtoss.pdf

If students can't break down 4 fives into its component parts, play PPW Solitaire and PPW Bingo (slides 27 & 28)

Regions support commutativity (4x5=5x4)
and support mental flexibility with number, such as factorization in algebra

MULTIPLICATIVE THINKING -THE 'AREA' MODEL

Use MAB to develop a written algorithm for multiplying ones by tens and ones, for example:



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \end{array}$$
^{1}

3 ones by 4 ones 12 ones.
Record ones with ones and ten with tens.
3 ones by 2 tens is 6 tens and 1 more ten ... 7 tens

ASSESSING MULTIPLICATIVE THINKING

The Learning Assessment Framework for Multiplicative Thinking

Level 1: Primitive Modelling
Level 2: Intuitive Modelling
Level 3: Sensing
Level 4: Strategy Exploring
Level 5: Strategy Refining
Level 6: Strategy Extending
Level 7: Connecting
Level 8: Reflective Knowing

- SCAFFOLDING NUMERACY IN THE MIDDLE YEARS Learning and Assessment Framework
- **https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/assessment/learnandassess.pdf**

Problems to Use to assess students' multiplicative thinking can be found at:

- https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/assessconduct.aspx

Distribute the booklets

Make sure that you have downloaded, printed and read through the selected tasks in either:

- Assessment Task Booklet: Option 1 (pdf - 216.75kb)
- Assessment Task Booklet: Option 2 (pdf - 630.32kb)

Students should use the same booklet for all of the tasks. The booklets therefore need to be issued, collected and re-issued until all tasks are completed. Tasks need to be completed in the order they appear in the Task Booklet.

www.education.vic.gov.au/studentlearning/teachingresources/maths

BUTTERFLY HOUSE…

Some children visited the Butterfly House at the Zoo.
They learnt that a butterfly is made up of 4 wings, one body and two feelers.

While they were there, they made models and answered some questions.

For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

_____ wings

BUTTERFLY HOUSE …		
TASK:	**RESPONSE:**	**SCORE**
a.	No response or incorrect	**0**
	Correct (28 wings, 7 bodies, 14 feelers)	**1**
b.	No response or incorrect	**0**
	Correct (4 butterflies)	**1**
c.	No response or incorrect	**0**
	Partially correct with some indication of multiplicative thinking (eg, multiplication algorithm attempted),or correct with little/no working, or correct but evidence of additive thinking, eg, 98+98+98+98	**1**
	All correct (392 wings, 98 bodies, 196 feelers) with evidence of multiplicative thinking, eg, algorithm applied correctly or efficient computation strategies such as doubling or renaming (eg, 400-8 for 4x98)	**2**
d.	No response or incorrect	**0**
	Correct (6 butterflies) but working and/or explanation indicative of additive thinking (eg, make-all, count all strategy), or incorrect with some indication that the task has been understood in terms of multiplication or division	**1**
	Correct (6 butterflies) with clear explanation in terms of other body parts, eg, “Can’t be 7 because not enough feelers”	**2**

www.education.vic.gov.au/studentlearning/teachingresources/maths

SCAFFOLDING NUMERACY IN THE MIDDLE YEARS

LAF Raw Score Translator Option 1

The following table is provided to enable teachers to locate students in terms of the **Learning and Assessment Framework for Multiplicative Thinking (LAF)** on the basis of their performance on the Assessment Tasks – Option 1.

To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 25 items altogether - 13 for Tables and Chairs, 4 for Butterfly House, 3 for Tiles, Tiles, Tiles, 2 for Adventure Camp, and 3 for Stained Glass Windows).

Total Score	LAF Level	Level Description
41-47	**8**	Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems
35-40	**7**	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent and proportion. May not be able to

Assessment for Common Misunderstandings

- https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/lvl3multi.aspx

Common Misunderstandings - Level 3 Multiplicative Thinking

Although most students at this Level have some knowledge of the multiplication facts to 100 and can perform simple multiplication and division procedures correctly, many rely on rote learning and/or a naïve, groups of understanding for multiplication based on repeated addition (often counting equal groups by ones). With little or no access to a broader range of ideas for multiplication they find it difficult to develop efficient mental strategies, and as a consequence, tend to rely on memorised procedures for multiplying and dividing larger whole numbers and decimals.

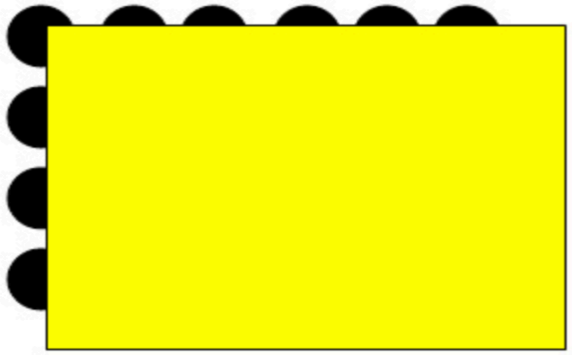
Example: Array and Region Question

Instructions

Put Card 2 on top of Card 1 so that it is completely covered, place in front of the student, and say, **"I'm going to show you what is underneath this card very quickly and I want you to tell me what you notice?"** Remove Card 2 and replace as quickly as possible. Note student's response (eg, "there are dots in rows").

Say, "**I'm going to show you some of the dots and hide the others**." As you say this, slide Card 2 over Card 1 so that only part of the array is shown as in the diagram below. Ask, **"Can you tell me how many dots there are altogether?"**

Note student's response



Example from The Advice Rubric

Counts by ones to get 24, may point to keep track of the hidden dots and/or use partial skip count (eg, counts by 3s to 18 then counts on top row by ones), experiences difficulty with A3 task (eg, may not realise cards need to be placed in same orientation)	May not be able to work with 4 or 6 as a countable unit, may not 'trust the count' or be able to keep track of a skip count • Use subitisation cards to check part-part-whole understanding for numbers to 10 and the extent to which students 'trust the count' for numbers 5 to 10 • Use ten-frames and 2-row bead frames to build knowledge of doubles, rehearse the use of doubling to more efficiently determine the number in an array (eg, for 5 rows of 6, think: double 6 is 12, double that again is 24 and 1 more 6 is 30)

Alternative approaches to teaching the multiplication facts

Teach students *the relationships between numbers*.

To start, teach students to learn the doubles of the numbers 1 to 9 (2,4,6,8,10, 12 etc) and then to build on this knowledge to find the other multiplication facts.

The doubles are important because they are the building blocks of a conceptual understanding of multiplication. Once children know their doubles, they can understand that three of anything is the same as a double plus one more group. Four of anything is the same as doubling a double. Five groups of anything is the same as half of ten groups of anything, and so on.

From Ocean, J., Sawatzki, C. & Ersozlu, Z. (2021). Spotting the Military Influence in Mathematics Education. *Australian Primary Mathematics Classroom,* 26 (1), 20-24.

- 0x – Anything multiplied by 0 is 0
- 1x – Anything multiplied by 1 is itself
- 2x – Think double
- 3x – Think double and add one more group
- 4x – Think double, double
- 5x – Think 10s facts, then halve the answer, or think 4 groups plus 1 group, etc.

The children learn mental flexibility when thinking like this.

- 6x – Think 5 groups and add one more group
- 7x – Think 5 groups plus a double, OR 3 groups plus 4 groups, OR 6 groups plus 1 group, etc
- 8x– Think four groups doubled, OR 10 groups take away a double, OR 5 groups plus 3 groups …etc
- 9x – Think 10 groups, then take away one group.
- 10x – Use place value knowledge - 10 groups of 2 is 2 tens (20), 10 groups of 3 is 3 tens (30) etc.

- The Eights and Nines Multiplication Facts don't require any fancy 'tricks'. The Eights can be thought of as 'the Tens take away a double' and the Nines as 'the Tens take away one group'.

The 'Deep Thought' Multiplication Facts Quiz

- **Instructions**
- In this game, the same questions are asked as in the ordinary Times Tables quiz, but the answers are different. The answers in this quiz re-frame the question.

- Teachers, change the starter question each time you start the game to give your students lots of different ways to think about numbers.
- I ask my students to stand in a large circle, and throw a large soft ball to the student I want to answer the first question. That student then throws the ball to the person they choose to answer their question etc.
- Students are allowed to say PASS if they don't know the answer, and throw the ball to someone else.

The game should be played frequently but only for about 10 mins at a time. Students focus intently on this game, thinking flexibly, which is very hard work (that's what we want!). It is better to do this for 10 minutes every day, than for half an hour once per week.

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Here is an example of a game:

The teacher asked the 'starter question': "3 tens *is the same as?"*

Student 1 responded, like this: "3 tens *is the same as* 10 threes. 10 threes *is the same as?*" (Then Student 1 passed the ball to Student 2.)

Student 2: "10 threes *is the same as* 5 threes, doubled. 5 threes doubled *is the same as?"*

Student 3: "5 threes doubled, *is the same as* 7 threes and 3 threes. That *is the same as?"*

Student 4: "7 threes and 3 threes is the same as 3 tens"

Student 5: "You can't say that, it's already been said (by the teacher).

Student 6: "Oh, ok. 7 threes and 3 threes is the same as 4 threes and 6 threes".

Student 7: "4 threes and 6 threes is the same as…

And so on….

When the group agrees they have run out of options to reframe the current question, the teacher chooses a new 'starter question'.

STUDENTS MIGHT NOT BE ABLE TO PLAY MULTIPLICATION TOSS IF THEY DON'T HAVE PART-PART-WHOLE NUMBER KNOWLEDGE.

HERE ARE TWO GAMES TO PLAY IF STUDENTS DON'T KNOW PART-PART-WHOLE –
E.G. IF THEY CAN'T DECOMPOSE THE NUMBERS FROM 3-9 EASILY

PPW Solitaire Friends-of-Ten Card game for Secondary School Students and Adults

- This game was created by teacher **Jenna Hewlett, Whyalla High School, South Australia**
- The mathematical point of this game is to teach students which numbers make up ten (The name of this concept is *part-part whole*)
- *Materials: I pack of cards per student*
- Give each student a pack of cards. They remove the J, Q, K , make the Ace worth 1.
- Students take the pack and shuffle.
- They lay out six cards in two rows and three columns
- They scan the six cards for side-by-side 'pairs of ten' .
- If they find one, they collect one point. If they can't find one, they subtract one point. (For younger students who don't understand negative numbers, they can play without subtracting until they have 5 points)
- Then they lay a card from the deck on top of one of the two 'pairs of ten', and scan again across the six cards for pairs of ten. This time, they can pair the "new" card with any of the other five cards. And so on…
- You can modify this game to play "Friends of 7", "Friends of 8", etc. so that you revise all the Part-Part-Whole Knowledge of the numbers 2 to 9.
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Bingo Friends-of-Ten Dice Game for Secondary School Students (also useful for Grades 5 & 6)

- This game was created by teacher **Jenna Hewlett, Whyalla High School, South Australia**
- The mathematical point of this game is to familiarise students with part-part-whole knowledge of the numbers 2-6; to give them practice at drawing up tables, and to think about probability.
- *Materials: 2 six sided dice per student*
- Aim: to help students to visualise the combination of 2 numbers. Each student rolls 2 six-sided dice (each of two different colours) 20 times and records the results in a 6x6 table with the numbers 1-6 on both horizontal and vertical axes. Eg a green 4 and a red 2 is recorded separately to a red 4 and a green 2.
- Then students copy from the board two 3x3 grids (Bingo chart) with nine numbers from 2-12 written in the boxes (teachers: choose these numbers randomly). Students are asked, "what do you think the chance is of getting your numbers, from adding the rolls of 2 dice?". Students roll their 2 dice and add them, then cross off the sum if it happens to be on their bingo chart. When they have 1 empty square left they "quack", then when they cross off the last number they call Bingo. (Teachers; have a winner, first runner up and second runner up).
- Students create one empty 3x3 grid. This time they choose which numbers to put in the grid. (Teachers: ask them to think about which sums of 2 numbers are most likely). Teacher rolls dice and calls out the numbers and their sum. Winners call out Bingo!
- Repeat Step 3 with another empty grid.
- Ask winners to share strategies and discuss
- Repeat Step 3 if you think it needs to happen for students to get the point that seven has most combinations (1,6, 6,1; 2,5 5,2; 3,4 4,3) next is six (1,5 5,1; 2,4 4,2 3,3) and eight, least is 2 (1,1).

Questions?